LEADING-EDGE BUCKLING UNDER THE COMBINED ACTION OF THERMAL STRESSES AND AERODYNAMIC LOADS

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ABSTRACT

An analysis is presented of the buckling of the leading edge of a thin solid wing, tapered linearly chordwise, under the combined action of spanwise thermal stress and aerodynamic load. Results are presented in the form of interaction curves for various degress of leading-edge bluntness.

INTRODUCTION

The buckling of a sharp leading edge due to thermal stresses arising from kinetic heating has been treated at Mansfield [1]. Biot [2], in a paper dealing with the divergence of supersonic wings, has analysed the leading-edge buckling of a wing under aerodynamic loading. In a further paper [3], Biot examines the effect of a combination of thermal and aerodynamic loading on the torsional stability of a wing.

Here, leading-edge buckling under the same combination of loads is treated, and interaction curves are given for a wing whose thickness tapers linearly chordwise for various degrees of leading-edge bluntness. Incidentally, an estimate of the effect of bluntness on leading-edge buckling under thermal stress alone is obtained and it can be seen that, while there is no singular behaviour for a sharp leading edge, the transition from perfectly sharp to a small but practical bluntness factor leads to a significant increase in the critical stress.

Leading-edge buckling, particularly for sharp leading edges and where thermal stresses are dominant, is a local effect and not likely to lead to a direct failure. In fact, its occurrence will relieve the compressive stresses at the leading edge and so decrease the losses of overall flexural and torsional stiffness due to thermal stress [4–7]. However, this factor is likely to be outweighed by the effect of the deformed leading edge on the pressure distribution and airflow over the wing. The former effect is described by Biot [2,3] and the latter would cause increased turbulence and hence aggravate the temperature gradients in the wing.

SYMBOLS

x, y		chordwise and spanwise coordinates
w		downward deflection
h		wing thickness
c		half chord length
h_0		thickness at midchord
h_1		thickness at leading edge
D		flexural rigidity
ν		Poisson's ratio
D_0		flexural rigidity at midchord
σ_y		spanwise stress
$\sigma_{y'}$		spanwise stress at leading edge
M		Mach number
ρ		air density
V		speed of sound
q		local lift on the wing
q_0	$=$	$2M^2 \rho V^2 / \sqrt{(M^2 - 1)}$
X, Y		nondimensional coordinates
∇^2	$=$	$\partial^2/\partial X^2 + \partial^2/\partial Y^2$
a	$=$	h_1/h_0
m	$=$	$1 - a$
k_1	$=$	$q_0 c^3 / D_0$
k_2	$=$	$h_0\, \sigma_{y'}\, c^2 / \pi^2 D_0$
l		spanwise half-wavelength
p	$=$	π/l
$r(X)$		chordwise variation of rigidity
$s(X)$		chordwise variation of thermal stress
$f(X)$		$\displaystyle\sum_{n=0}^{\infty} a_n X^n$ chordwise variation of deflection

$f_2(X), f_3(X)$ defined by Eq. (13)

A_2, A_3 arbitrary constants

$\phi_2, \phi_3, \psi_2, \psi_3$ defined by Eq. (14)

Δ = $\phi_2\psi_3 - \phi_3\psi_2$

k_1^*, k_2^* values of k_1, k_2, for which $\Delta = 0$

OUTLINE OF PROBLEM AND ASSUMPTIONS

The problem treated is that of the buckling of the leading edge of a wing of symmetrical double wedge cross section and infinite aspect ratio, flying at zero incidence and acted upon by both thermal and aerodynamic loadings. The assumption of zero incidence is made because it is the purpose of the present work to study the stability problem in isolation. The effect of a finite angle of attack is considered by Biot [2,3]. For the purpose of simplification of the analysis and calculations, the wing is assumed to be clamped along the midchord. The error arising from this assumption should be small because the trailing edge of the wing is stabilised by the aerodynamic load and so will not buckle. Further, in cases where the leading edge is fairly sharp and thermal stresses are dominant, buckling is confined to a region near the leading edge and the midchord has no tendency to deform. The spanwise thermal stresses are self-equilibrating over the wing cross section, compressive at the leading and trailing edges and tensile in the midchord region. They are here assumed to vary parabolically chordwise, thus embodying the general character of many practical distributions. The aerodynamic loading is taken to be that given by thin aerofoil theory—that is, normal to the middle surface of the wing and directly proportional to the local slope of the middle surface.

ANALYSIS

The spanwise middle surface stress σ_y must be self-equilibrating over the cross section of the wing, so that

$$\int_{-c}^{c} h\sigma_y \, dx = 0 = \int_{-c}^{c} xh\sigma_y \, dx \tag{1}$$

following the coordinate system and notation shown in Fig. 1. Thus if σ_y is taken to obey a parabolic law,

$$\sigma_y = \sigma_y{}'\{6(2 - m)(x/c)^2 - 4 + 3m\}/(8 - 3m) \tag{2}$$

The local lift on the wing is assumed to be given by thin aerofoil theory, that is

$$q = 2M^2 \rho V^2 \frac{\partial w}{\partial x} \bigg/ \sqrt{(M^2 - 1)} = q_0 \frac{\partial w}{\partial x} \qquad (3)$$

Now the differential equation to be satisfied by the buckled plate is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left\{ D \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w \right\} - (1 - \nu) \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 w}{\partial y^2}$$

$$= q_0 \frac{\partial w}{\partial x} - h\sigma_y \frac{\partial^2 w}{\partial y^2} \qquad (4)$$

where σ_y is taken to be positive in compression and D is given by

$$D = D_0 \left(1 - m|x/c| \right)^3$$
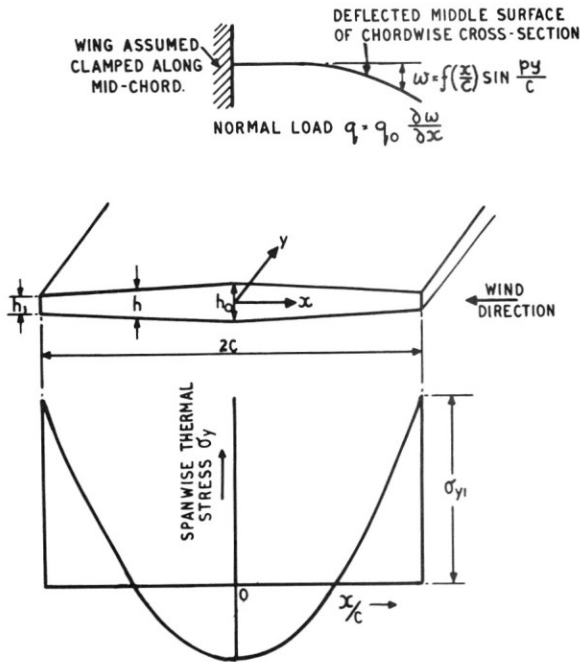

Figure 1.   Load systems and notation.

It is now convenient to introduce the parameters

$$X = |x/c|, \qquad Y = y/c$$

$$r(X) = D/D_0 = (1 - mX)^3$$

$$s(X) = h\sigma_y/h_0\,\sigma_y' = (1 - mX)\{6(2 - m)X^2 - 4 + 3m\}/(8 - 3m)$$

$$k_1 = q_0 c^3/D_0\,, \qquad k_2 = h_0\,\sigma_{y_1}c^2/\pi^2 D_0 \tag{5}$$

$$\nabla^2 = \partial^2/\partial X^2 + \partial^2/\partial Y^2$$

and the subscript 1 denotes d/dX. Thus, Eq. (4) becomes:

$$r\nabla^4 w + 2r^1\frac{\partial}{\partial X}\,(\nabla^2 w) + r^{11}\left(\frac{\partial^2 w}{\partial X^2} + \nu\,\frac{\partial^2 w}{\partial Y^2}\right) \mp k_1\,\frac{\partial w}{\partial X} + \pi^2 k_2 s\,\frac{\partial^3 w}{\partial Y^2} = 0 \tag{6}$$

However, since the midchord of the wing is assumed clamped, it is necessary to consider only the forward half of the wing and so the negative sign before the term in k_1 applies.

Now, taking a buckling mode of the form

$$w = f(X)\sin pY \tag{7}$$

where the spanwise half-wavelength

$$l = \pi/p,$$

substituting into Eq. (6) and collecting terms, the following ordinary differential equation is obtained for f:

$$(1 - mX)^3\,f^{1111} - 6m(1 - mX)^2\,f^{111} + 2(1 \;-\!\!-\; mX)\{3m^2$$

$$- p^2\,(1 - mX)^2\}f^{11} + \{6mp^2(1 - mX)^2 - k_1\}f^1 + p^2(1$$

$$- mX)[p^2(1 - mX)^2 - 6\nu m^2 - \pi^2 k_2\{6(2 - m)X^2 - 4$$

$$+ 3m\}/(8 - 3m)]f = 0 \tag{8}$$

If now, a power series is assumed for f,

$$f = \sum_{n=0}^{\infty} a_n X^n \tag{9}$$

and is substituted into Eq. (8), the following recurrence relation for the a_n's is obtained:

$$
\begin{aligned}
a_{n+4} = {}& \frac{3m(n+2)a_{n+3}}{n+4} \\[6pt]
&+ \frac{\{2p^2 - 3m^2(n+2)(n+1)\}a_{n+2}}{(n+4)(n+3)} \\[6pt]
&- \frac{\{6mp^2(n+1) - m^3(n+2)(n+1)n - k_1\}a_{n+1}}{(n+4)(n+3)(n+2)} \\[6pt]
&- \frac{p^2\{p^2 - 6m^2(n^2+n+\nu) + \pi^2 k_2(4-3m)/(8-3m)\}a_n}{(n+4)(n+3)(n+2)(n+1)} \\[6pt]
&- \frac{mp^2\{2m^2(n^2-1+3\nu) - 3p^2 - \pi^2 k_2(4-3m)/(8-3m)\}a_{n-1}}{(n+4)(n+3)(n+2)(n+1)} \\[6pt]
&- \frac{3p^2\{m^2p^2 - 2\pi^2 k_2(2-m)/(8-3m)\}a_{n-2}}{(n+4)(n+3)(n+2)(n+1)} \\[6pt]
&+ \frac{mp^2\{m^2p^2 - 6\pi^2 k_2(2-m)/(8-3m)\}a_{n-3}}{(n+4)(n+3)(n+2)(n+1)}
\end{aligned} \tag{10}
$$

where

$$ n = 0, 1, 2, 3, \ldots $$

and by definition

$$ a_{-3} = 0 = a_{-2} = a_{-1} $$

Thus the function $f(X)$ can be determined in terms of the four arbitrary constants a_0, a_1, a_2, a_3.

The boundary conditions at the midchord are:

$$ w = 0 = \partial w/\partial x $$

giving

$$ a_0 = 0 = a_1 \tag{11} $$

while at the leading edge

$$ \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0 = \frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x \partial y^2} \quad \text{on } x = c $$

or

$$ f^{11}(1) - \nu p^2 f(1) = 0 = f^{111}(1) - (2-\nu)p^2 f^1(1) \tag{12} $$

It is now convenient to introduce the functions:

$$f_2(X) = X^2 + \sum_{n=4}^{\infty} a_{2,n}X^n$$

$$f_3(X) = X^3 + \sum_{n=4}^{\infty} a_{3,n}X^n$$

and write

$$f(X) = A_2 f_2(X) + A_3 f_3(X) \tag{13}$$

so that it becomes possible to determine all the $a_{2,n}$'s and $a_{3,n}$'s from Eq. (10). Thus writing

$$\phi_k = f_k^{11}(1) - \nu p^2 f_k(1)$$

$$= k(k-1) - \nu p^2 + \sum_{n=4}^{\infty} \{n(n-1) - \nu p^2\} a_{k,n}$$

$$\psi_k = f_k^{111}(1) - (2-\nu)p^2 f_k^{1}(1) \tag{14}$$

$$= k\{(k-1)(k-2) - (2-\nu)p^2\} + \sum_{n=4}^{\infty} n\{(n-1)(n-2)$$

$$- (2-\nu)p^2\} a_{k,n}$$

$$k = 2, 3$$

the condition for buckling is

$$\Delta = \phi_2\psi_3 - \phi_3\psi_2 = 0 \tag{15}$$

It is clear that Δ is a function of five variables

$$\Delta = \Delta(m, p, k_1, k_2, \nu)$$

Throughout the numerical work Poisson's ratio is taken to have the value 0.3. If particular values of m and k_1 are chosen, then for every positive value of p there will exist a series of values of k_2 for which

$$\Delta = 0$$

Suppose k_2^* is the numerically smallest of these values, then the required solution is obtained by varying p until k_2^* becomes a true minimum. Owing to the complexity of the functions involved, this minimum value of k_2^* cannot be determined analytically, so that a numerical procedure must

be adopted. It is clearly also possible to choose particular values of m and k_2 and to find the minimum value of k_1^* as a function of p. In practice, one of these procedures is usually more convenient than the other, depending on the particular values under consideration.

The numerical work was performed using the IBM 7090 located at the Weapons Research Establishment, Salisbury, South Australia.

RESULTS AND DISCUSSION

Figure 2 shows the interaction between the critical aerodynamic load parameter k_1 and the critical thermal stress parameter k_2 for a number of values of the bluntness factor a. For $a = 0$, the critical value of k_2 in the absence of k_1, and the limit of the critical values of k_1 in the absence of k_2 are indicated. As is explained by Biot [2], this limit is finite even though buckling can occur at zero load for $a = 0$.

Because of the poor convergence of the series for small a, the smallest value for which an interaction curve is obtained is $a = 0.05$. For this case,
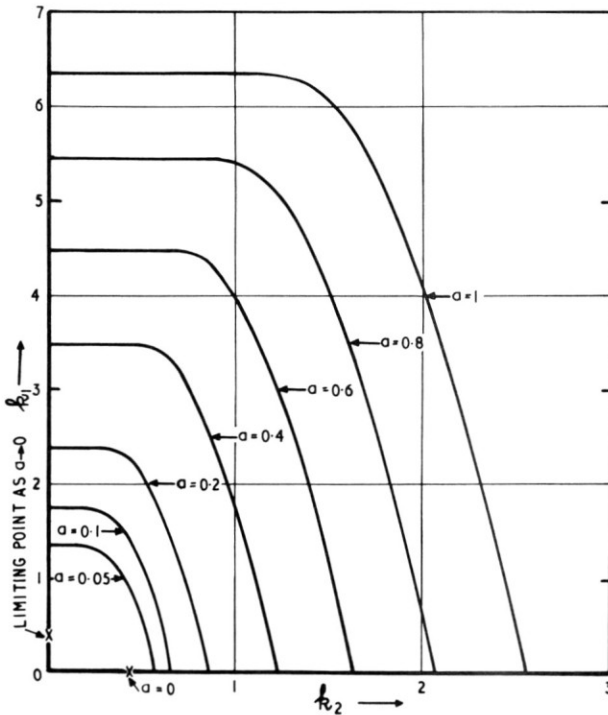

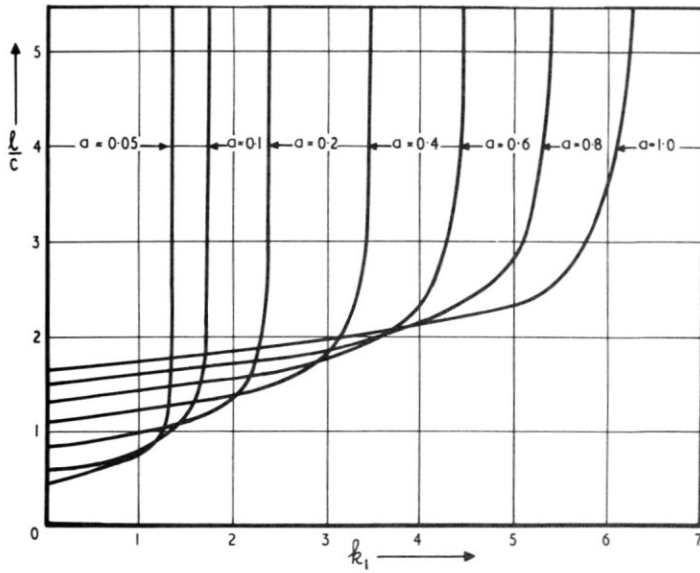Figure 2. The interaction between critical thermal and aerodynamic load parameters.

Figure 3. The variation of spanwise wavelength with critical
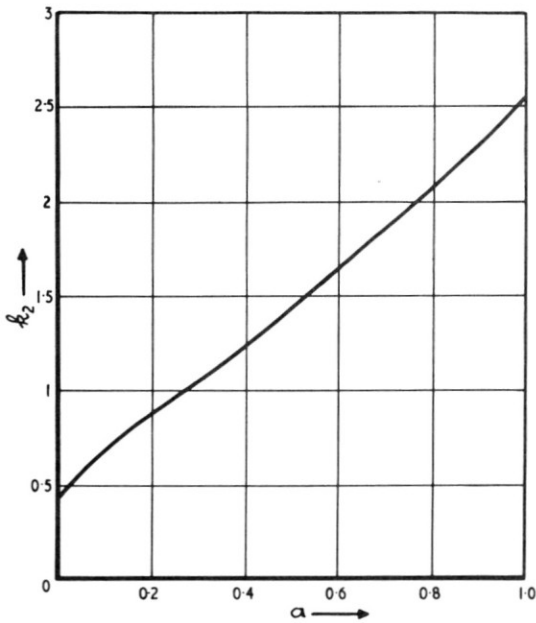aerodynamic load parameter.

Figure 4. The variation of critical thermal stress with bluntness
factor in the absence of aerodynamic load.

more than 500 terms were required for each series. Although interaction
curves for smaller values of a and in particular their limit as $a \to 0$ would
be of interest, it is unlikely that a leading edge designed for operation at
high supersonic speed would have a bluntness factor less than 0.05.

In Fig. 3 the spanwise wavelength is plotted against the critical values of
k_1, the corresponding values of k_2 being understood to be those given by
Fig. 2. It can be seen that wavelength tends to decrease as the leading edge
becomes sharper and when the thermal stresses are dominant, the effect of
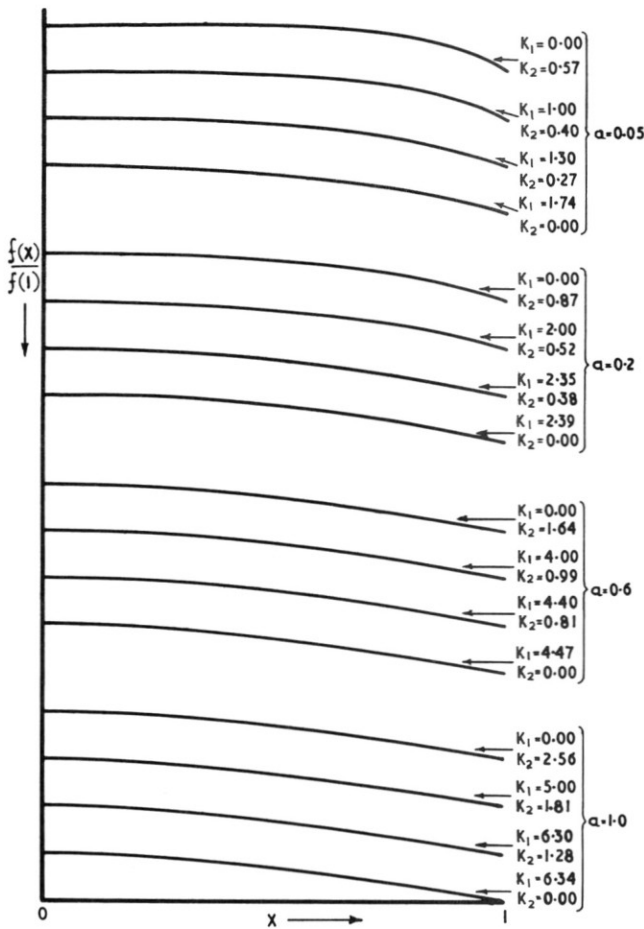increasing aerodynamic load being to increase the wavelength.

Figure 5. Some typical chordwise mode shapes.

Calculation shows that in many practical cases k_1 is less than unity and thus, as can be seen from Fig. 2, the aerodynamic load has little effect on the critical thermal stress. Exceptions to this would occur for very thin wings (thickness to chord ratio of 1 per cent or less) or for wings flying at high speed and low altitude. Figure 4 shows the increase in critical thermal stress with bluntness factor in the absence of aerodynamic load. As predicted by Mansfield [1], although there is no singular behaviour for a sharp leading edge, the effect of bluntness is marked. For a bluntness factor of 0.1, the critical thermal stress increases by about 50 per cent over the value for a sharp leading edge.

It is of interest to obtain an estimate of the numerical value of temperature difference at which buckling would occur. If the relationship between k_2 and a shown in Fig. 4 is treated as linear, the temperature difference for a steel wing at buckling may be written:

$$T \doteq 16 \times 10^4 \left(\frac{h_0}{2c}\right)^2 (1 + 6a)$$

indicating that buckling may frequently be an important factor in the design of a leading edge.

Figure 5 shows the chordwise deflection in the initially buckled state for a range of critical values of the parameters. Some estimate of the limitations imposed by the assumption of a clamped midchord may be obtained from this figure. The expected tendencies are observed, namely, that the blunter the wing, the farther from the leading edge do the buckles extend, and that for a given bluntness the effect of increasing the proportion of aerodynamic load is to enlarge the buckled area.

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ACKNOWLEDGEMENT

COMMENTARY

PROF. DR. A. VAN DER NEUT (*Technical University, Delft, Netherlands*): In reply to Prof. Hoff's question: The need for 500 terms of the series originates from the chosen location of the origin of the coordinates x. The origin lies at mod-chord where the deflections are extremely small, whereas the interesting part of the deflection curve is near the leading edge. If the origin would have been chosen at the leading edge, I presume, the solution could have been obtained with a small number of terms.

I might put forward another question. The very interesting aspect of this investigation is the inclusion of the aerodynamic effect as it occurs with stationary air flow. One could think of the possibility of leading-edge flutter. Then the time-dependent aerodynamic forces of unstationary air flow come into the picture. However, we have learned from the present communication that the aerodynamic forces do affect the static instability to a slight extent only. My question is: May it then be conjectured that there exists no risk of leading-edge flutter in practice?

REPLY

The author would like to thank Professor van der Neut for his interesting comments. While it is true that better convergence might well have been obtained using a different origin of coordinates, the very large number of terms was only required when the leading edge was fairly sharp ($a = 1/20$) and a genuine singularity exists for a perfectly sharp edge. A different choice of origin would have increased the complexity of the analysis and the numerical work in that the buckling determinant would have become 4×4 rather than 2×2.

In regard to the question about flutter—it is obviously very difficult to draw firm conclusions from the present example. Furthermore, a simple extension of the analysis to include vibration of the leading edge would not include the effect of unsteady airflow over the wing. I think that this question could probably best be resolved through an experimental program, possibly backed by an approximate analysis.